# Event Tree Analysis

Best Practices in Dam and Levee Safety Risk Analysis

Part A – Risk Analysis Basics

Chapter A-5

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US Army Corps of Engineers®

U.S. DEPARTMENT OF THE INTERIOR BUREAU OF RECLAMATION

DEPARTMENT OF ENERGY FEDERAL ENERGY REGULATORY COMMISSION

TVA

# Objectives

- Define event tree terminology and rules
- Demonstrate common applications

# Outline of Topics

- Structure
- Terminology
- Calculations
- Construction

# Key Concepts

- **Event Tree Analysis** is an inductive modeling technique that uses Boolean logic to evaluate a sequence of events
- Frequently used concepts and techniques include
    - Conditional – Probability depends on an event that has occurred
    - Intersection – Used to multiply probabilities
    - Mutually Exclusive – Used to sum probabilities
    - Partitioning – Used to discretize continuous functions
    - Consistent Percentile – Used to combine uncertainties

# Event Tree Analysis

- A model for estimating risk
- Depicted by an event tree
- Used to decompose and discretize a complex sequence of events

- Improves understanding of potential failure modes
- Alternative models
  - Fault tree analysis
  - Stochastic simulation

**Initiating Event**

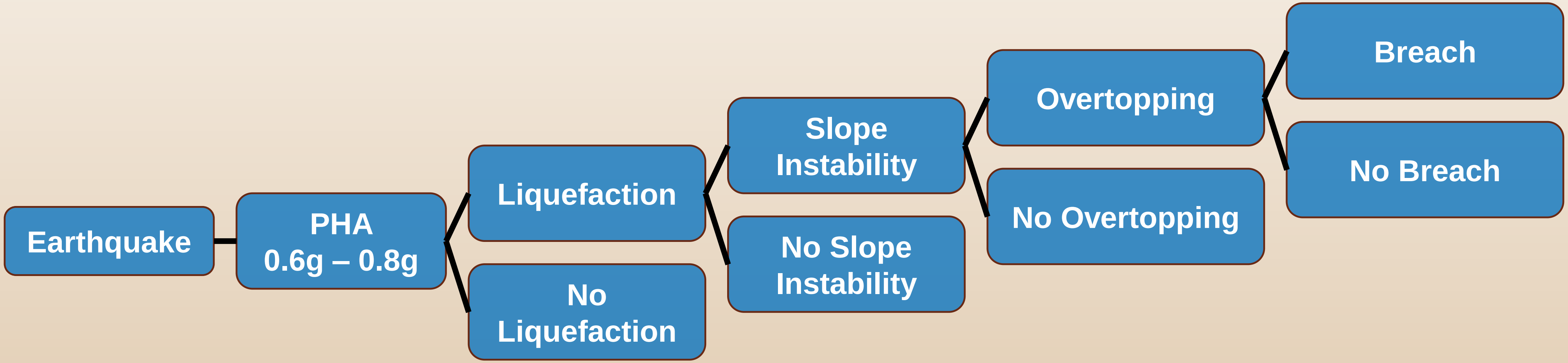
**Forward Looking (Inductive) Logic**
- **Chronological**
- **Causal Chain**

**Consequences**

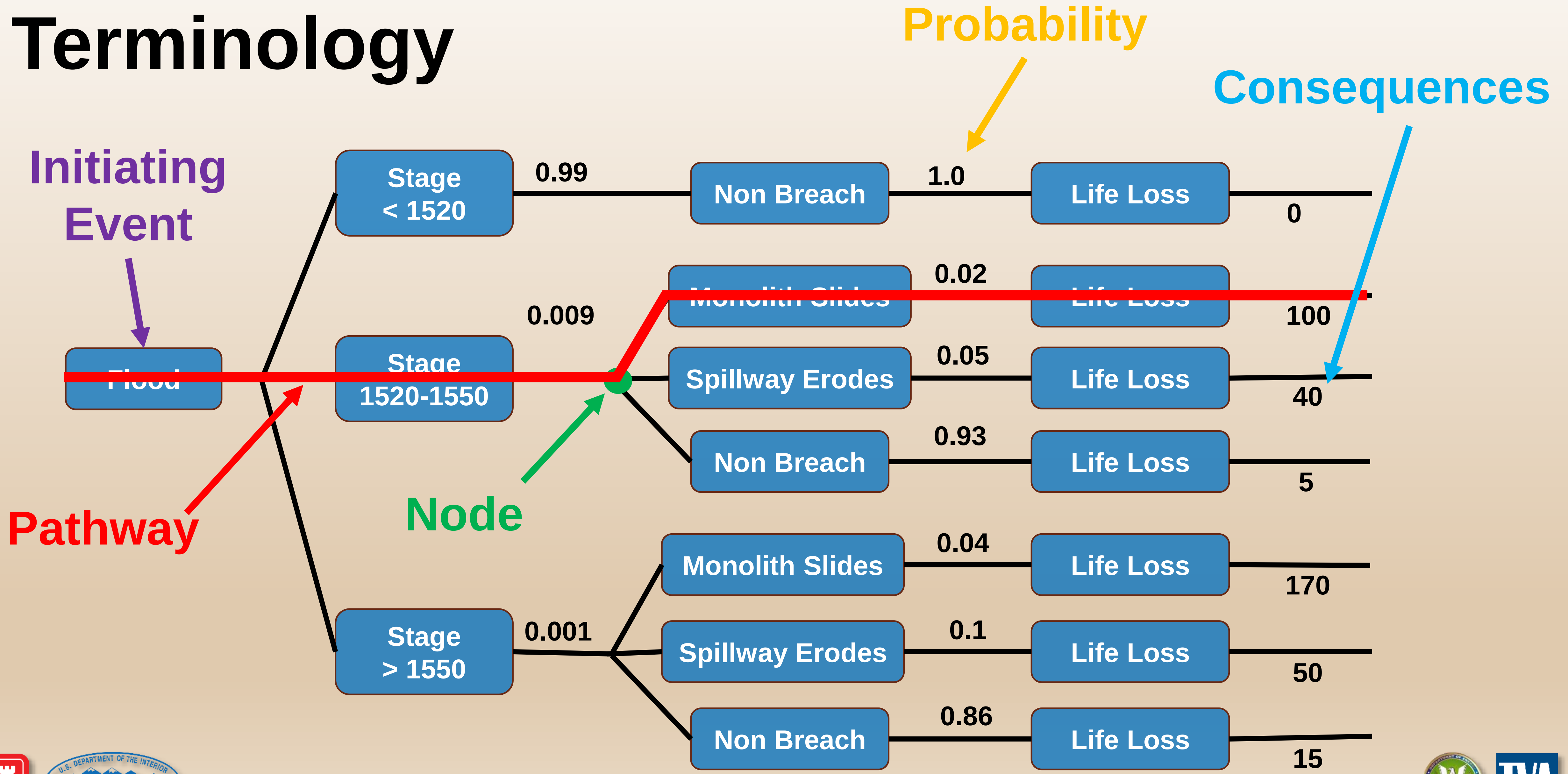
# Example

- Verbal PFM description
  - In a given year, an earthquake occurs with a peak horizontal acceleration between 0.6g and 0.8g. The ground motion triggers foundation liquefaction which causes instability of the upstream embankment slope. The resulting slope failure lowers the crest of the dam to a level below the reservoir pool. Overtopping of the lowered crest ensues causing erosion and breach of the dam.
- Key events
  - Earthquake occurs with PHA between 0.6g and 0.8g
  - Foundation liquefaction is triggered
  - Upstream slope instability lowers the crest
  - Overtopping erodes the lowered crest
  - Breach occurs

# Possible Event Tree

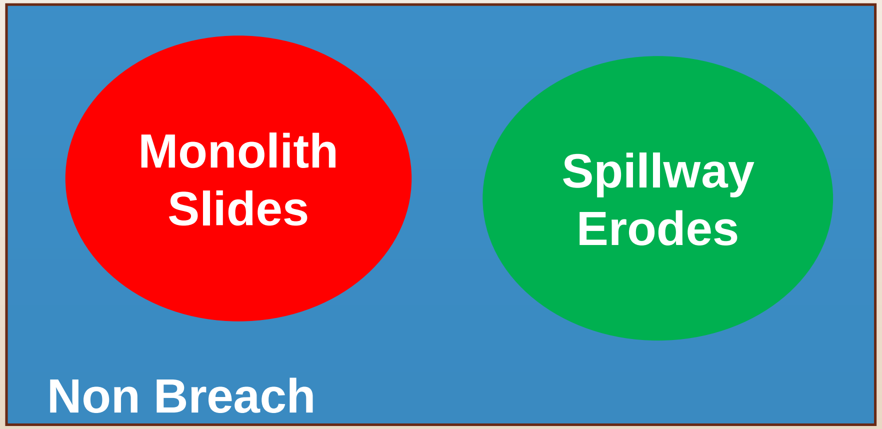


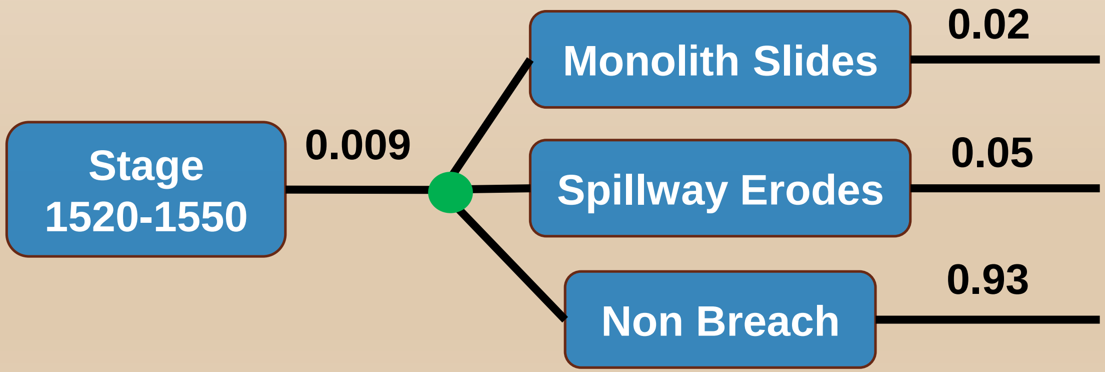
# Terminology



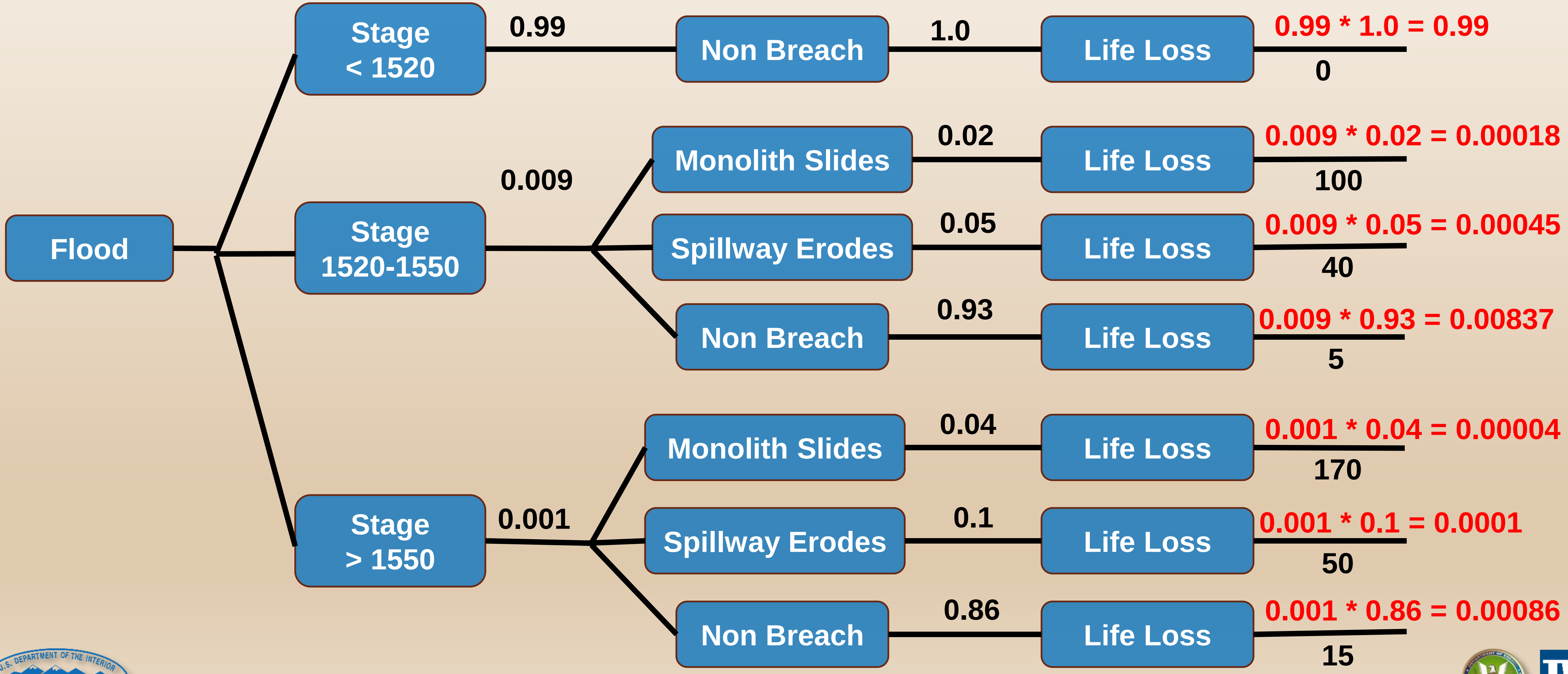
# Rules and Math

- Branches must be mutually exclusive
  - Only one outcome can occur
  - Probabilities across branches can be summed
- Probabilities must be conditional
  - Probability of an event depends on all events along pathways to the left
  - Probabilities along pathways can be multiplied
- Branches must be collectively exhaustive
  - The sum of probabilities across all branches must equal one

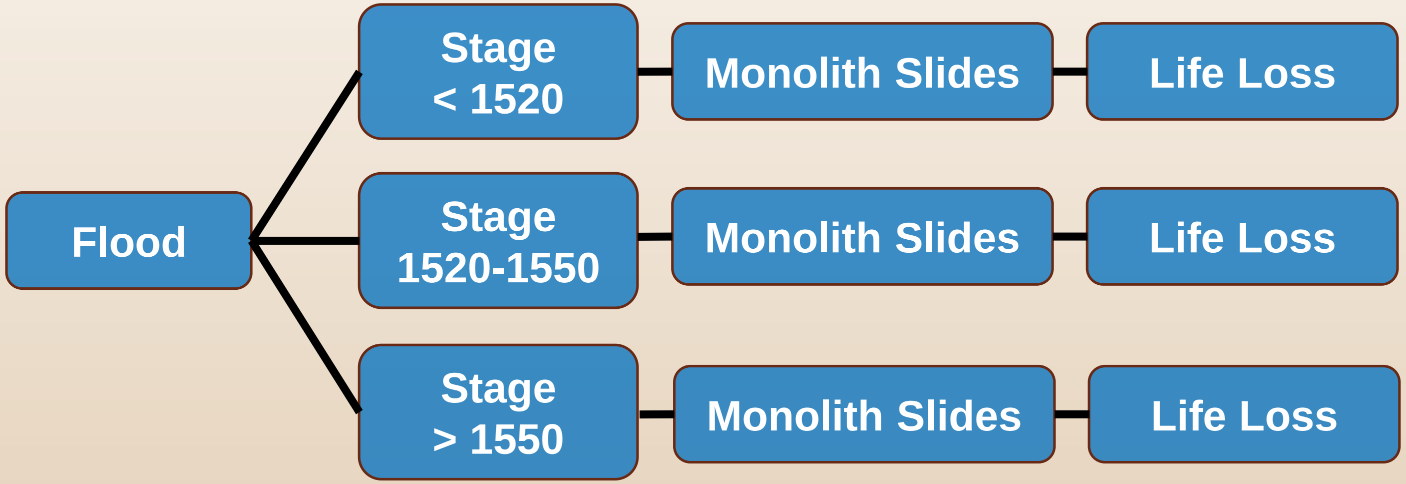


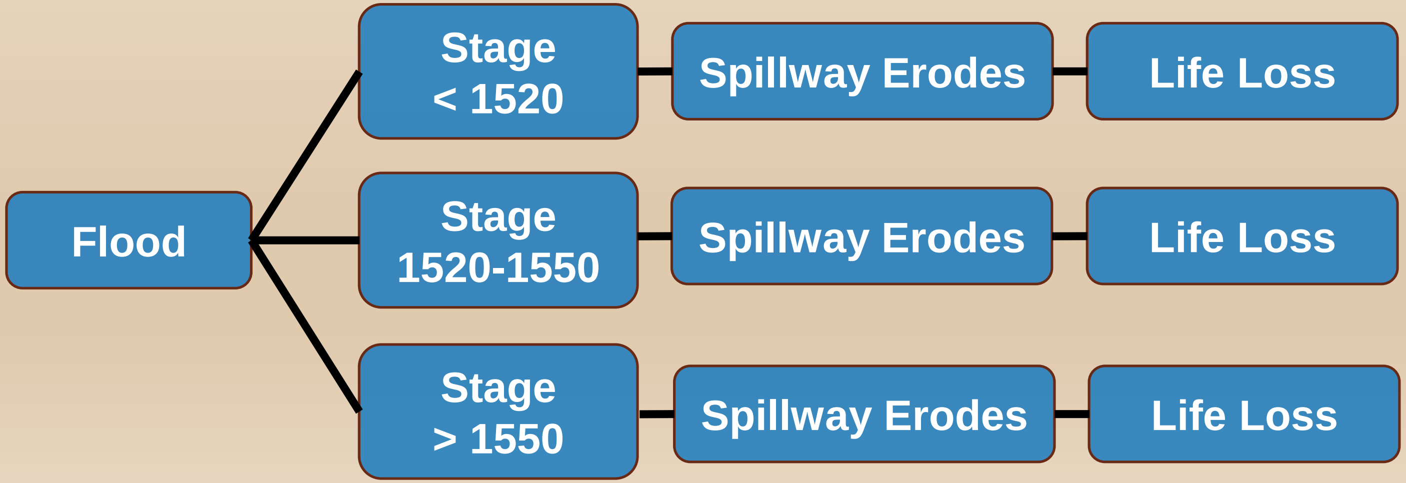
# Single Tree Format



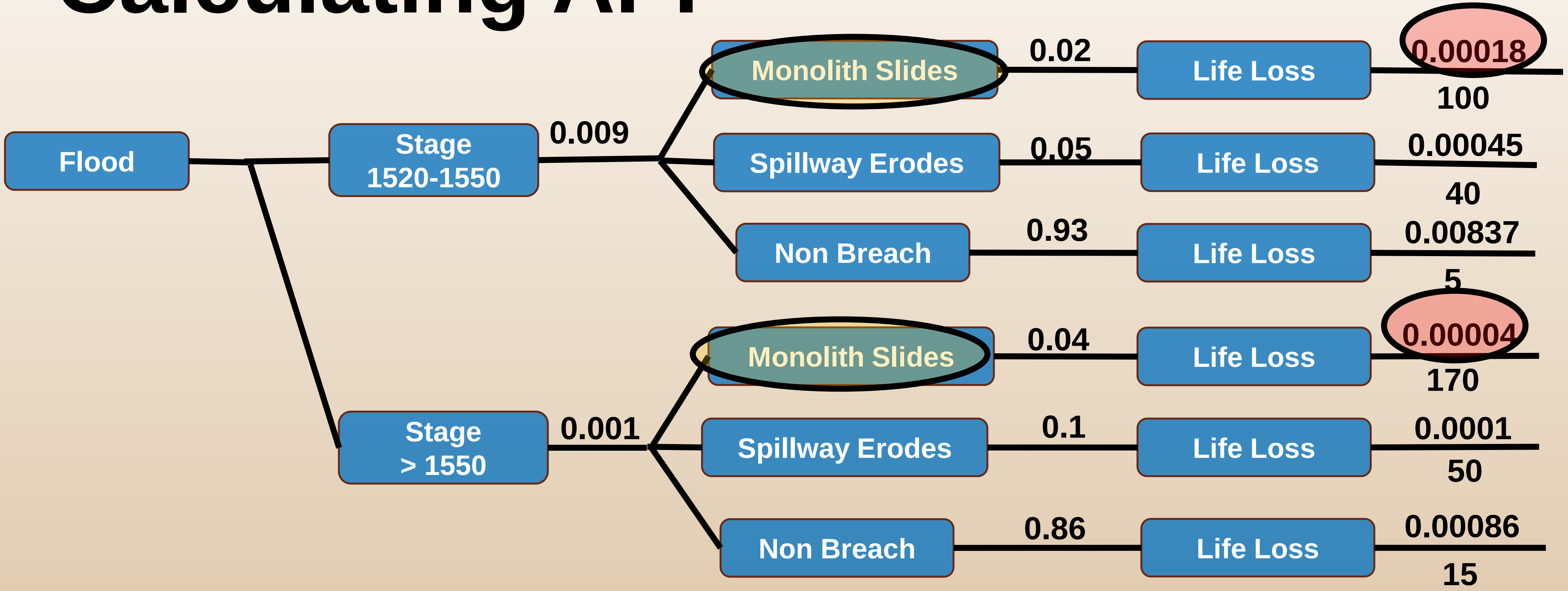
# Separate Potential Failure Mode Trees




**Non breach event tree not shown**



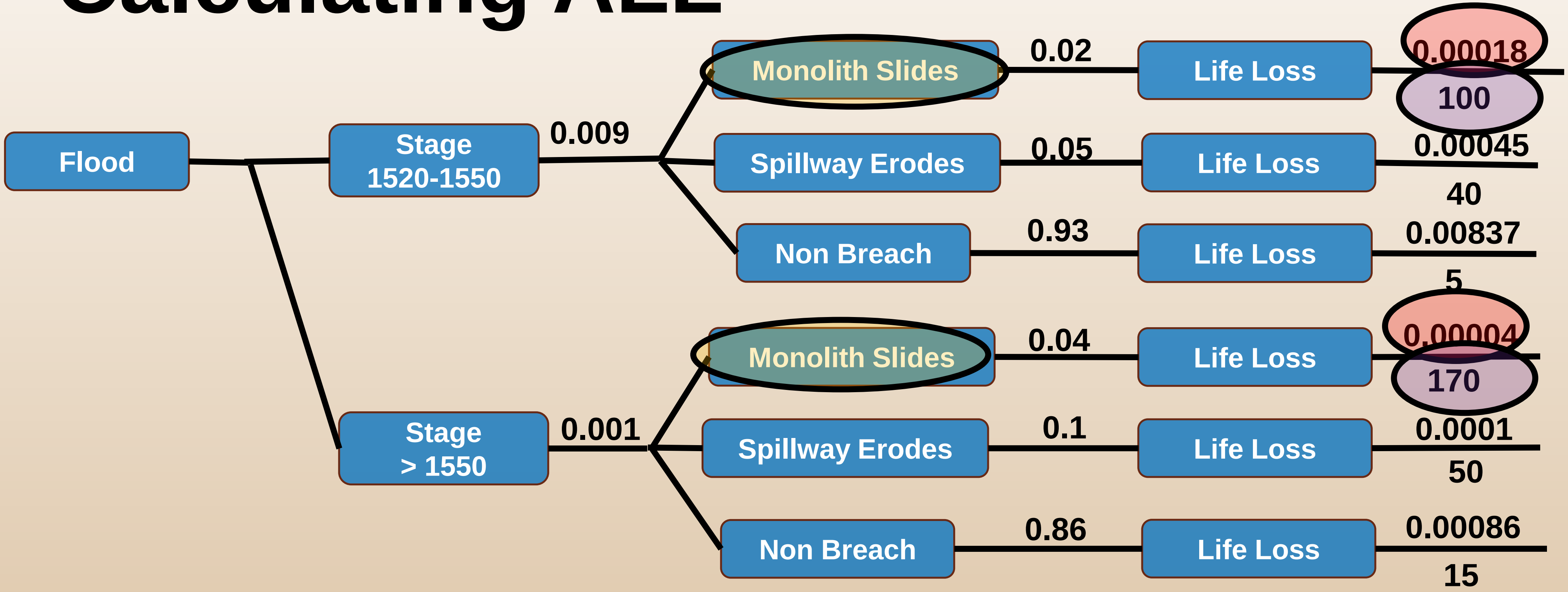
# Calculating APF




P(Event A) = Sum of end branch p values for all pathways that contain Event A

**APF(Monolith Sliding) = 0.00018 + 0.00004 = 0.00022**

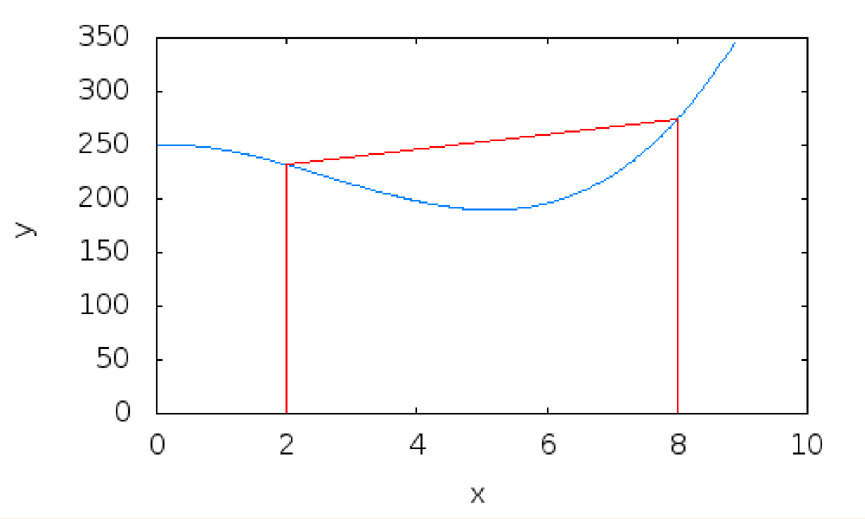
# Calculating ALL




E(C | Event A) = Sum of end branch p*c values for all pathways that contain Event A

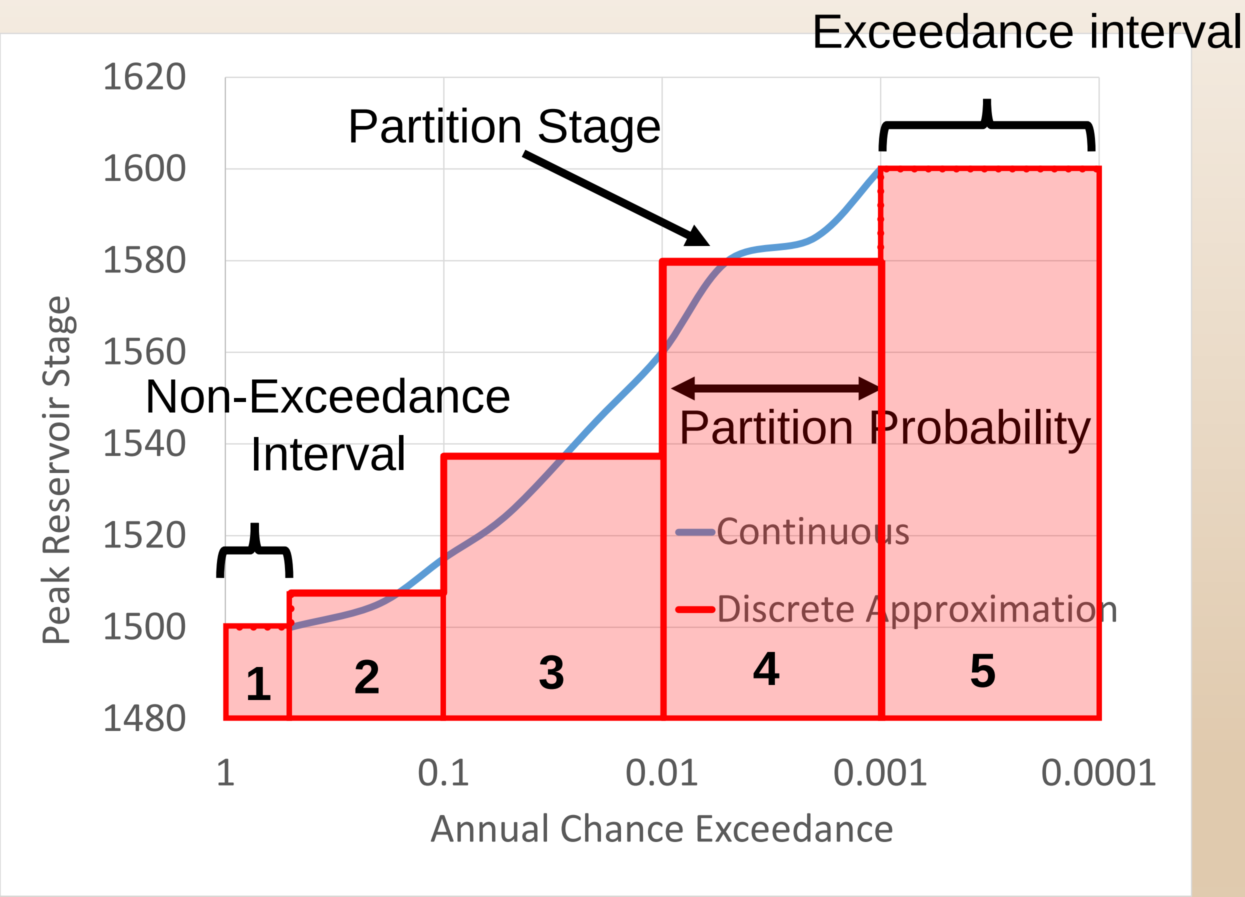
ALL(Monolith Sliding) = 0.00018 (100) + 0.00004 (170) = 0.0248

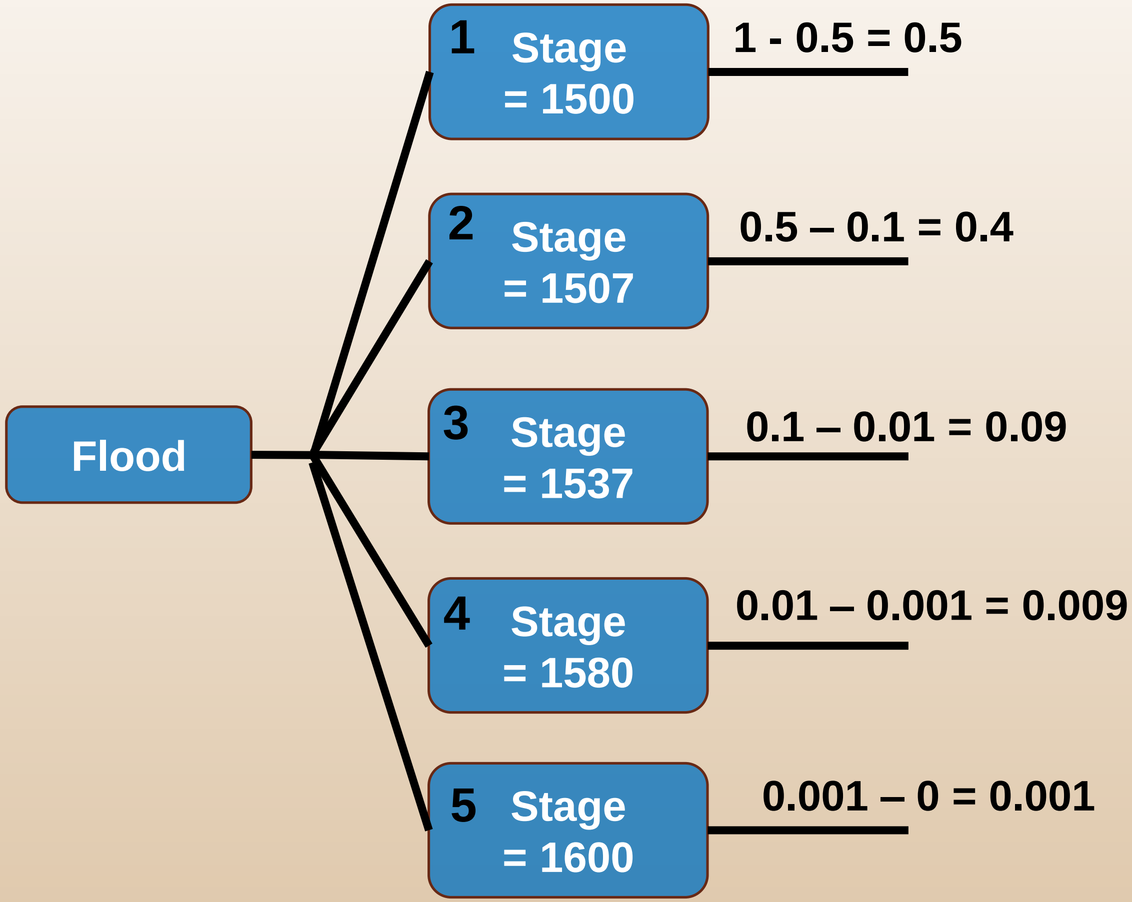
# Partitioning




- Tree branches are discrete
- Input functions are continuous
- Analogous to Simposon’s rule for integration
- Numerical precision
  - Number of partitions (more is better)
  - Location of partitions (capture shape changes)
- Can generate intervals manually or automatically
- Intervals can be regular or irregular spacing

# Example

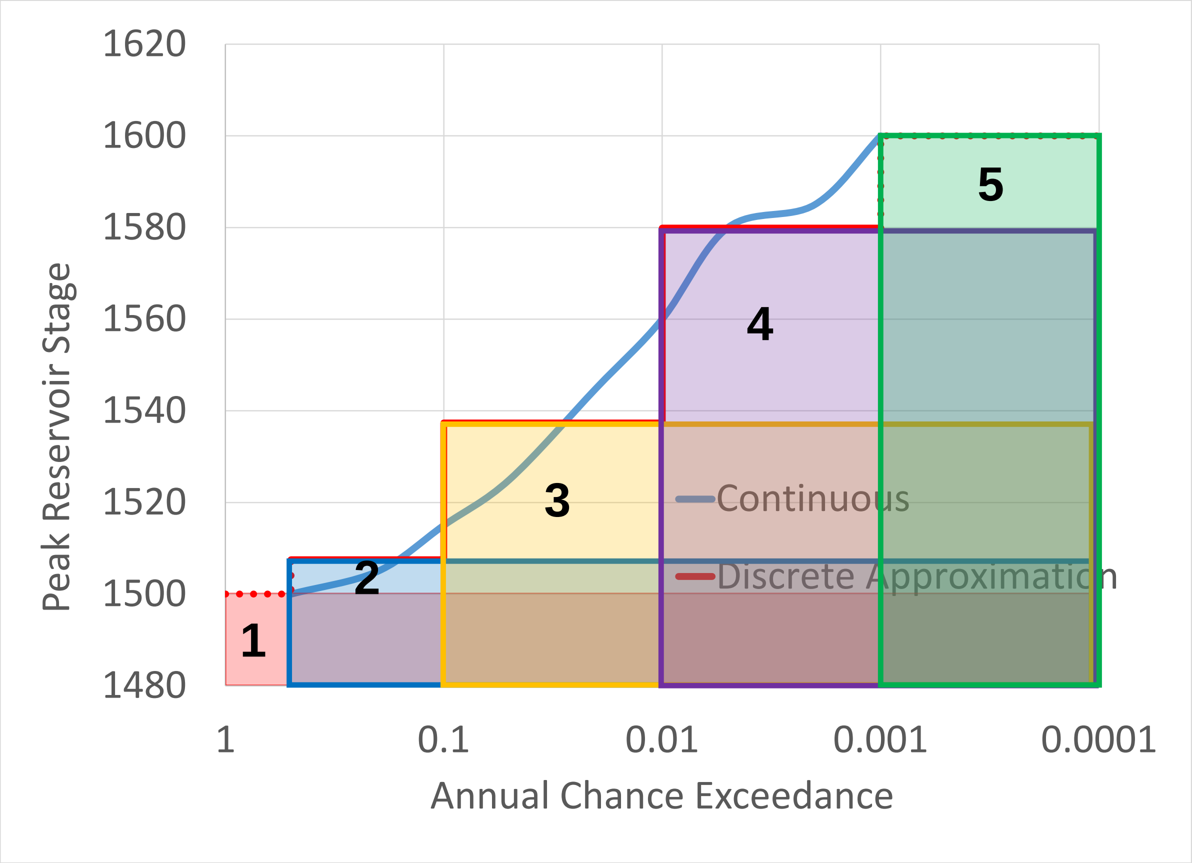


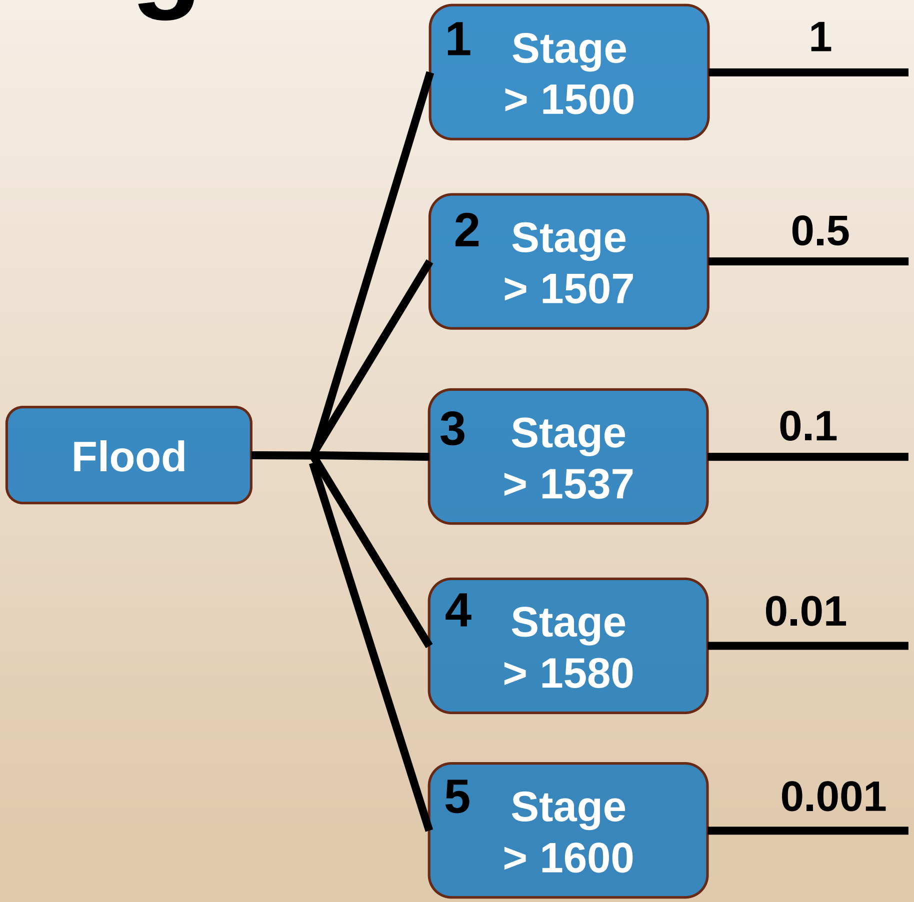




$\sum$ (area under the curve)= 1

These partitions are mutually exclusive

# Avoid Double Counting



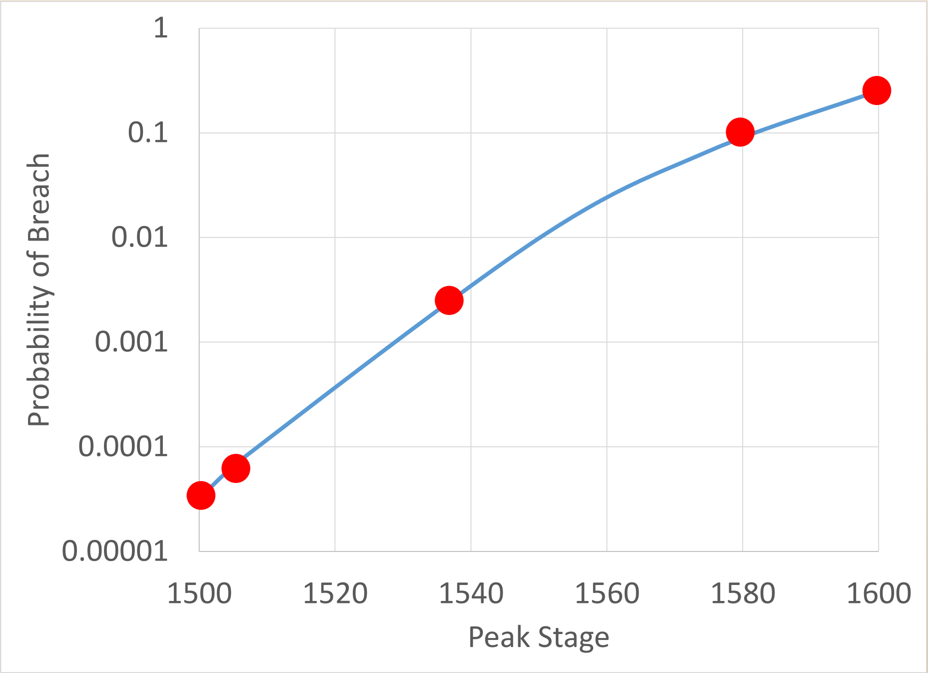



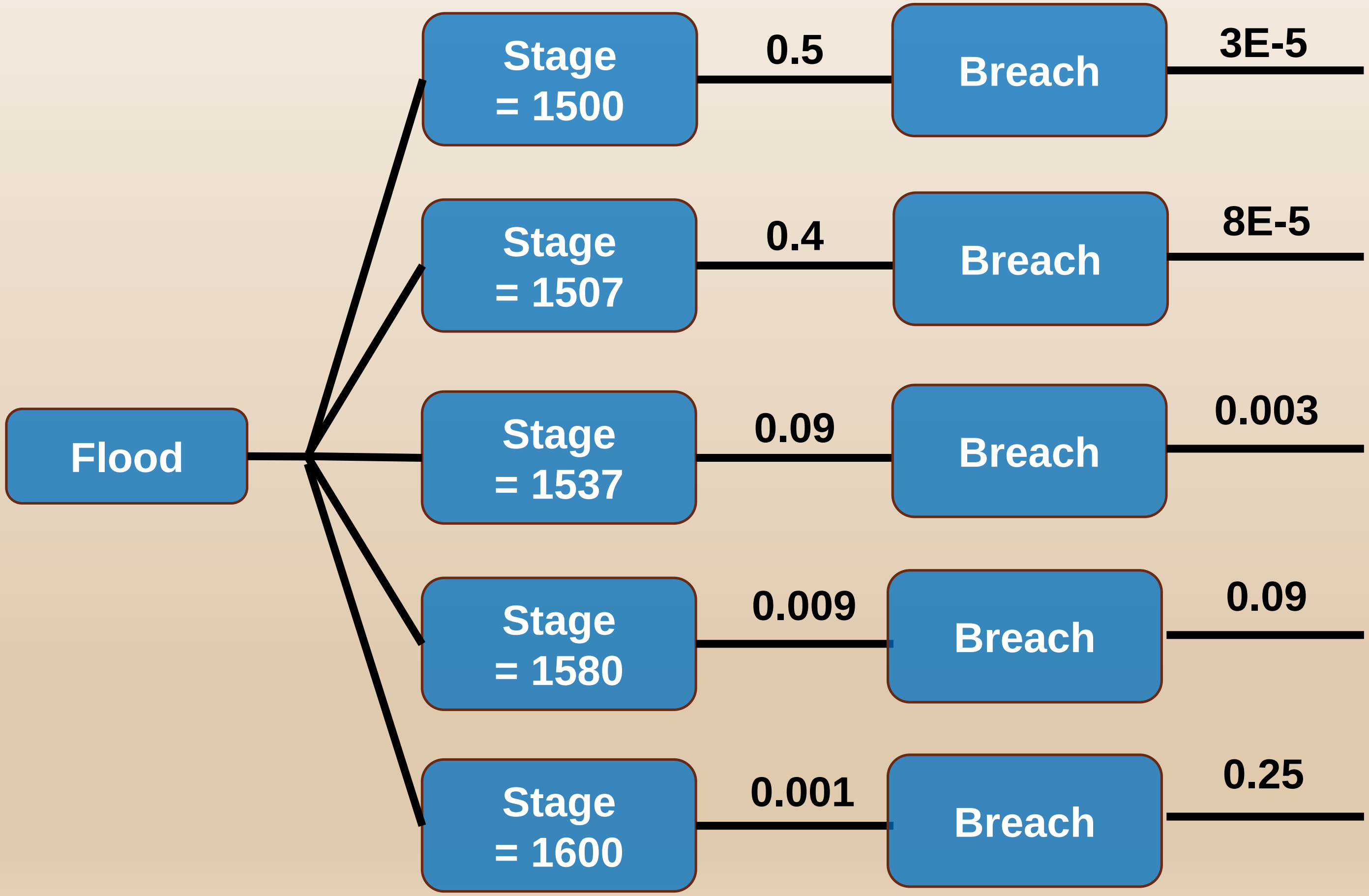

These partitions are not mutually exclusive

$\sum > 1$, not good

Do Not Use Exceedance Probabilities

# System Response Curves



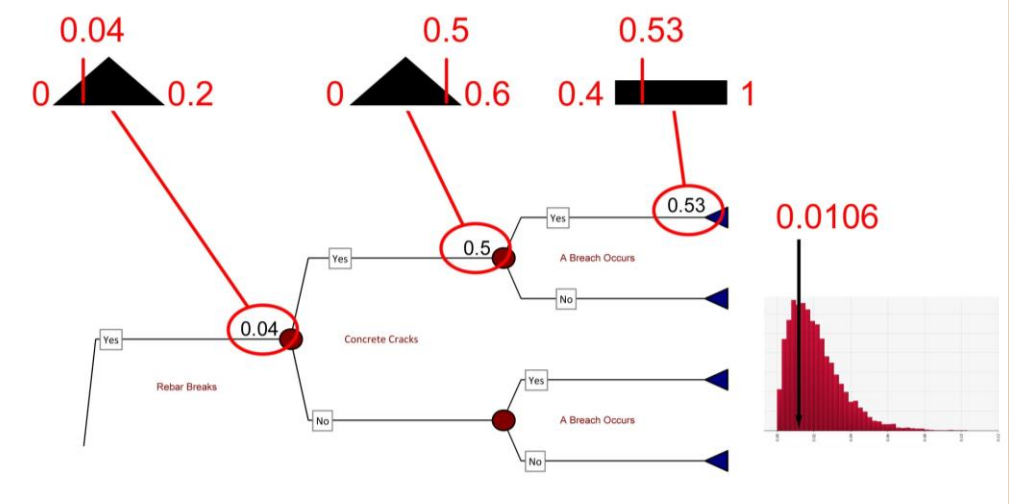



# Variable Transformation

- Peak stage is typically used as the independent variable to combine the hazard, system response, and consequence functions
  - Peak stage defined as a function of AEP
  - SRP and consequences defined as a function of peak stage
- Other variables might be
  - More convenient – Probability of failure as a function of overtopping depth
  - Better indicator – Consequences as a function of peak outflow
- Event tree calculations can be set up to perform and apply these transformations
  - Overtopping depth defined as stage minus top of levee
  - Peak outflow defined as function of flood AEP

# Monte Carlo Analysis

- Branch probability estimates and consequences can be modeled with uncertainty
- Monte Carlo analysis can be used to combine these uncertainties to obtain the uncertainty distribution for APF and ALL

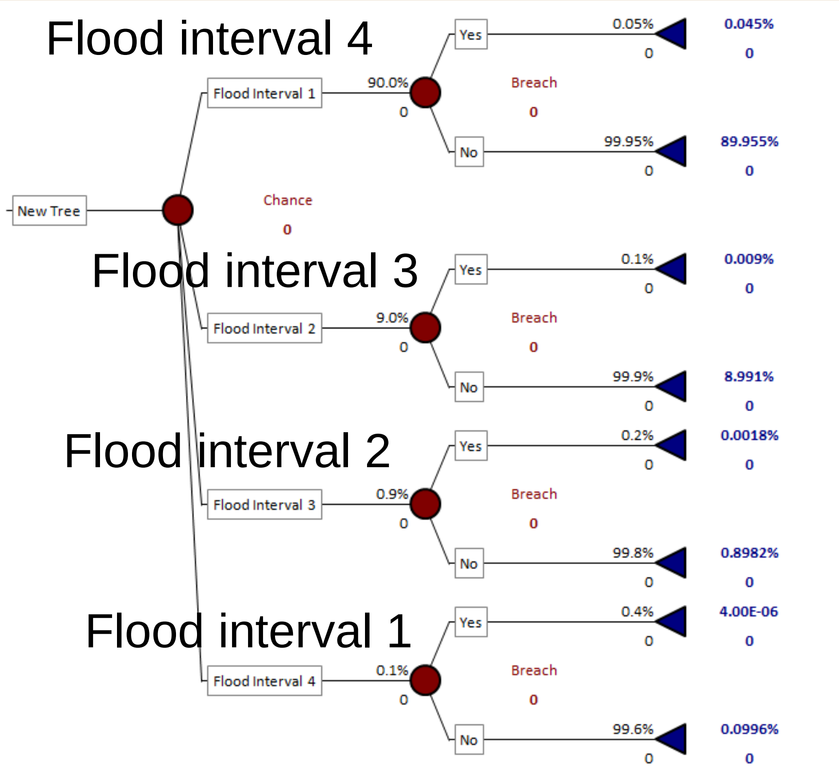


# Distribution of Sums and Products

- Because event tree math is additive and multiplicative
  - The mean AFP and mean ALL can be estimated by using the means of the input distributions
- Can become problematic in other models with operations that are not strictly additive or multiplicative
  - Use the mean of the output distribution from a monte carlo simulation
- The distribution of AFP and ALL will typically trend toward a normal or log normal distribution because of the central limit theorem

# Curve Sampling

- Independent sampling of each load partition can generate physically impossible samples

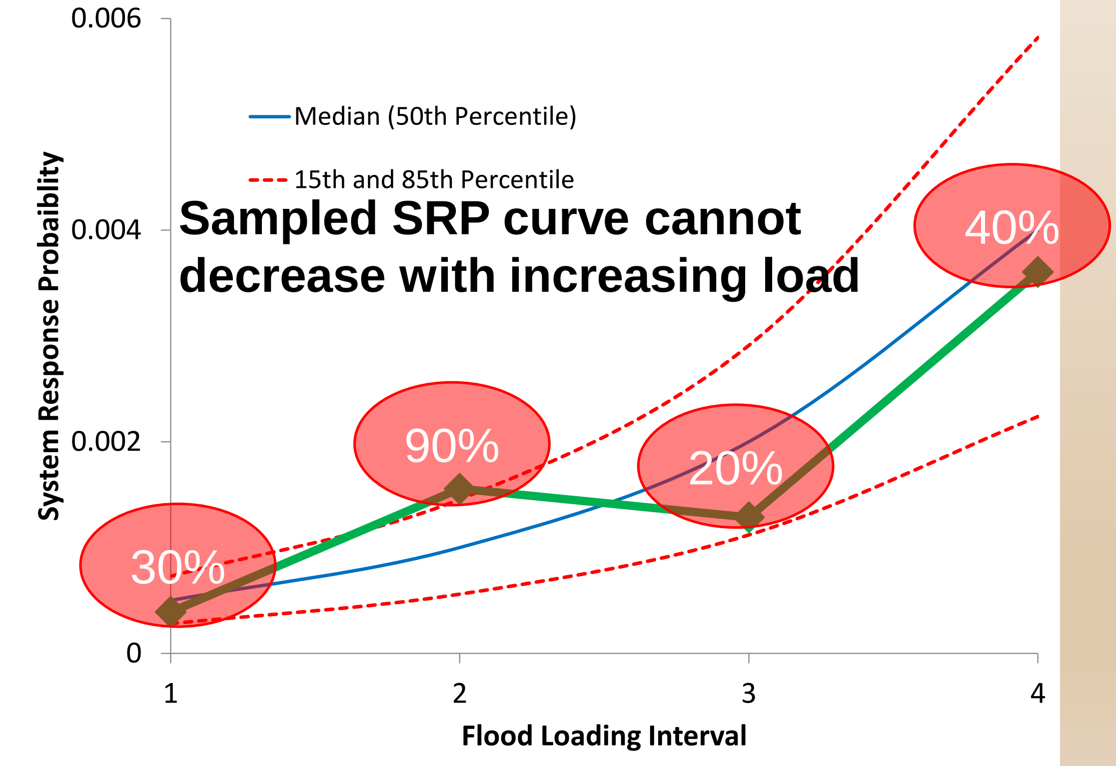



**0.3**

**0.9**

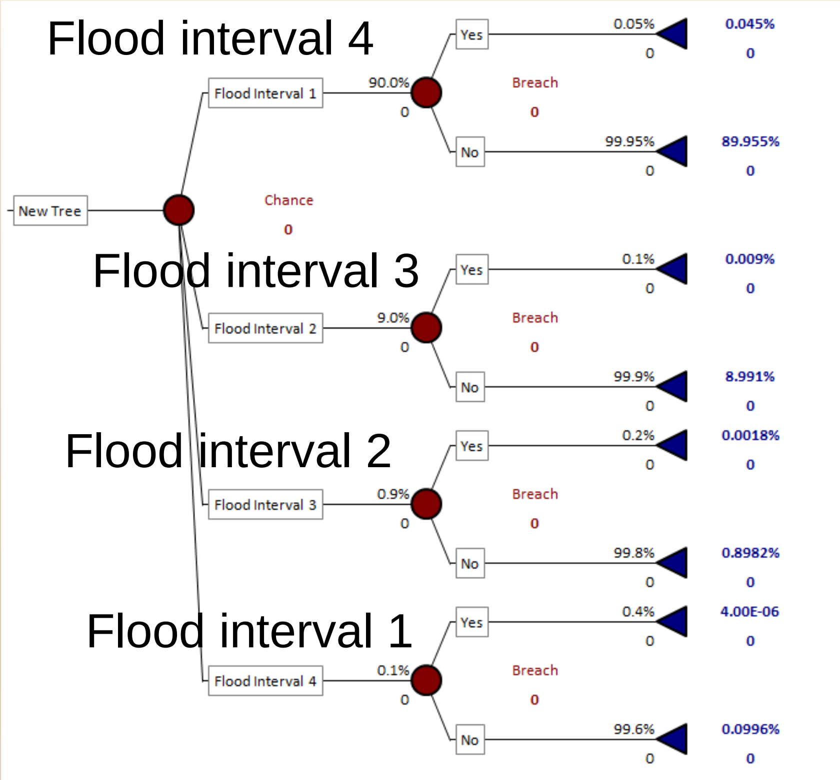
**0.2**

**0.4**

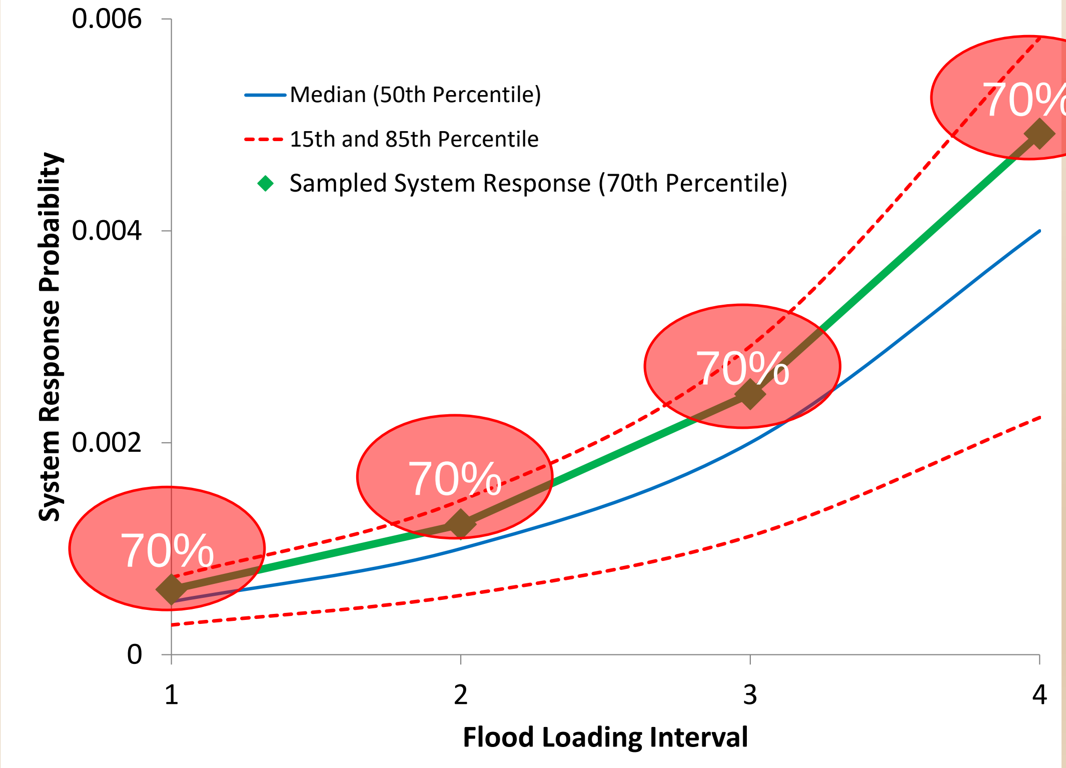


# Consistent Percentile Sampling

- Sample a single percentile and apply to all loading partitions



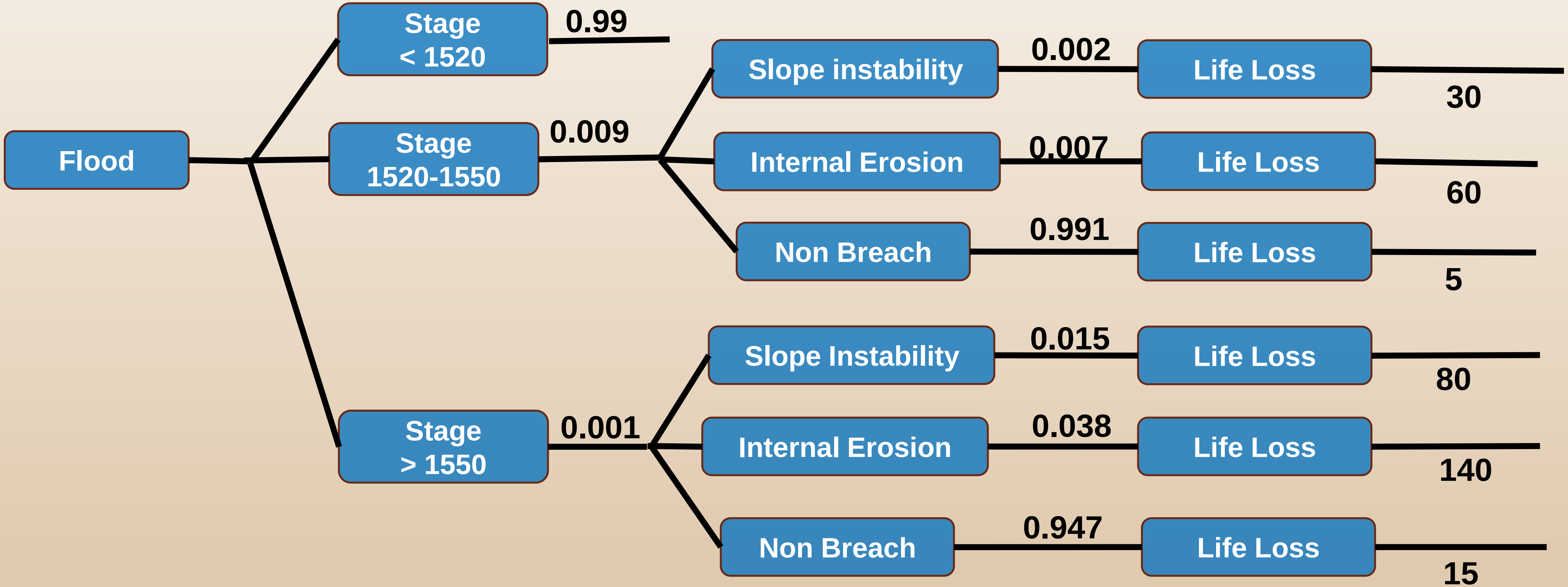

**0.7**



"Risk taking is inherently failure prone. Otherwise, it would be called sure thing taking."

-Jim McMahon

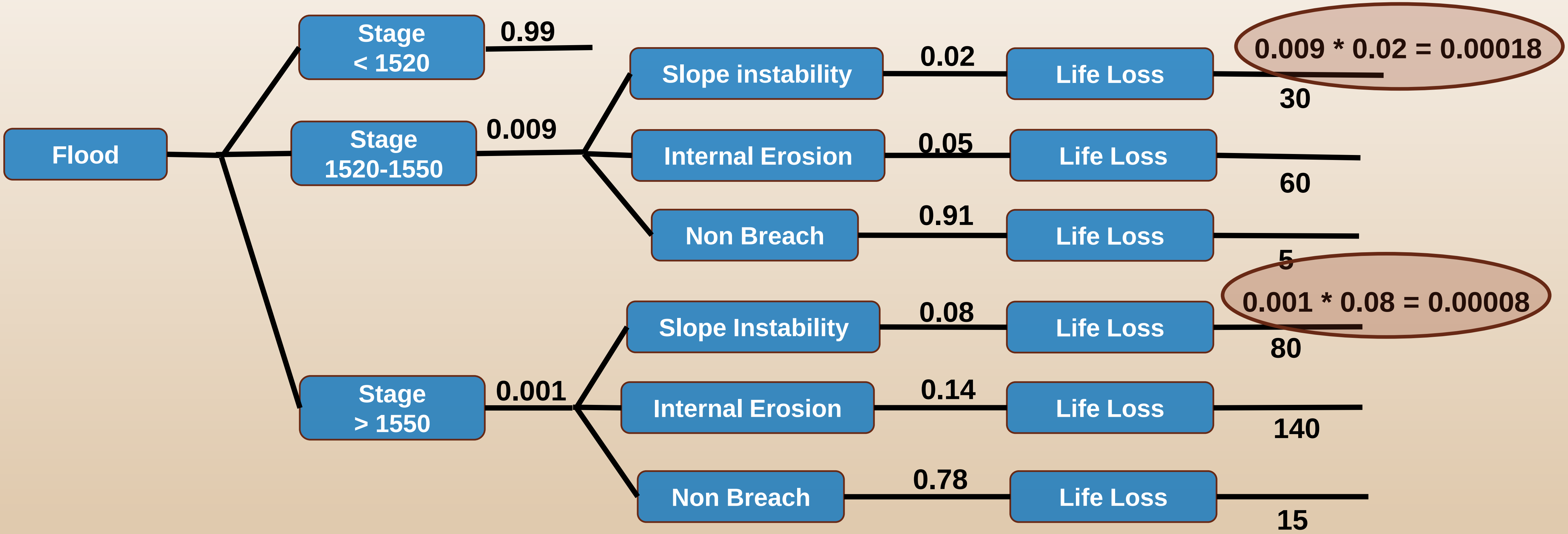
# Exercise




Calculate APF for slope instability

Calculate ALL for slope instability

# Solution




Calculate APF for slope instability = 0.00018 + 0.00008 = 0.00026

Calculate ALL for slope instability = (0.00018 * 30) + (0.00008 * 80) = 0.0118